In a nutshell: Approximating integrals using interpolating polynomials

In this topic, we assume that the x or t values are equally spaced. We will assume that equal spacing equals h. The derivative is being approximated at x_k or t_k, as applicable.

One-interval backward formulas

Trapezoidal rule

$$\int_{t_k-h}^{t_k} y(t)\mathrm{d}t \approx \frac{y(t_k)+y(t_{k-1})}{2}h = \frac{y(t_k)+y(t_k-h)}{2}h$$

Half-Simpson's rule

$$\int_{t_k-h}^{t_k} y(t)\mathrm{d}t \approx \frac{5y(t_k)+8y(t_{k-1})-y(t_{k-2})}{12}h = \frac{5y(t_k)+8y(t_k-h)-y(t_k-2h)}{12}h$$

Four-point backwards rule

$$\int_{t_k-h}^{t_k} y(t)\mathrm{d}t \approx \frac{9y(t_k)+19y(t_{k-1})-5y(t_{k-2})+y(t_{k-3})}{24}h = \frac{9y(t_k)+19y(t_k-h)-5y(t_k-2h)+y(t_k-3h)}{24}h$$

Composite rules

Divide the interval $[a, b]$ into n sub-intervals of width h so that $x_k = a + kh$ so that $x_0 = a$ and $x_n = b$

Composite trapezoidal rule

$$\int_a^b f(x)\mathrm{d}x \approx h\left(\frac{1}{2}f(x_0)+\left(\sum_{k=1}^{n-1} f(x_k)\right)+\frac{1}{2}f(x_n)\right)$$

Composite Simpson's rule assuming *n* is even

$$\int_a^b f(x)\mathrm{d}x \approx h\left(\frac{1}{3}f(x_0)+\frac{4}{3}f(x_1)+\frac{2}{3}f(x_2)+\frac{4}{3}f(x_3)+\cdots+\frac{2}{3}f(x_{n-2})+\frac{4}{3}f(x_{n-1})+\frac{1}{3}f(x_n)\right)$$

Composite 4-point centered rule

$$\int_a^b f(x)\mathrm{d}x \approx h\left(-\frac{1}{24}f(x_{-1})+\frac{1}{2}f(x_0)+\frac{25}{24}f(x_1)+\left(\sum_{k=2}^{n-2} f(x_k)\right)+\frac{25}{24}f(x_{n-1})+\frac{1}{2}f(x_n)-\frac{1}{24}f(x_{n+1})\right)$$